Precise Measurement of the $\pi^+ \rightarrow \pi^0 e^+ \nu$ Branching Ratio

D. Počanić,[1, *] E. Frlež,[1, †] V. A. Baranov,[3] W. Bertl,[2] Ch. Brönnimann,[2] M. Bychkov,[1] J. F. Crawford,[2] M. Daum,[2]
N. V. Khomutov,[3] A. S. Korenchenko,[3] S. M. Korenchenko,[3] T. Kozlowski,[4] N. P. Kravchuk,[3] N. A. Kuchinsky,[3]
W. Li,[1] R. C. Minehart,[1] D. Mzhavia,[3] B. G. Ritchie,[6] S. Ritt,[2] A. M. Rozhdestvensky,[3] V. V. Sidorkin,[3]
L. C. Smith,[1] I. Supek,[7] Z. Tsamalaidze,[5] B. A. VanDevender,[1] Y. Wang,[1] H.-P. Wirtz,[2, ‡] and K. O. H. Ziock[1]

[1]*Department of Physics, University of Virginia, Charlottesville, VA 22904-4714, USA*
[2]*Paul Scherrer Institut, Villigen PSI, CH-5232, Switzerland*
[3]*Joint Institute for Nuclear Research, RU-141980 Dubna, Russia*
[4]*Institute for Nuclear Studies, PL-05-400 Swierk, Poland*
[5]*Institute for High Energy Physics, Tbilisi State University, GUS-380086 Tbilisi, Georgia*
[6]*Department of Physics and Astronomy, Arizona State University, Tempe, AZ 85287, USA*
[7]*Rudjer Bošković Institute, HR-10000 Zagreb, Croatia*
(Dated: 9 July 2004)

Using a large acceptance calorimeter and a stopped pion beam we have made a precise measurement of the rare $\pi^+ \rightarrow \pi^0 e^+ \nu \, (\pi_\beta)$ decay branching ratio. We have evaluated the branching ratio by normalizing the number of observed π_β decays to the number of observed $\pi^+ \rightarrow e^+ \nu \, (\pi_{e2})$ decays. We find the value of $\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu)/\Gamma(\text{total}) = [1.036 \pm 0.004 \, (\text{stat.}) \pm 0.004 \, (\text{syst.}) \pm 0.003 \, (\pi_{e2})] \times 10^{-8}$, where the first uncertainty is statistical, the second systematic, and the third is the π_{e2} branching ratio uncertainty. Our result agrees well with the Standard Model prediction.

The rare pion beta decay, $\pi^+ \rightarrow \pi^0 e^+ \nu$ (branching ratio $R_{\pi\beta} \simeq 1 \times 10^{-8}$), is one of the most basic semileptonic electroweak processes. It is a pure vector transition between two spin-zero members of an isospin triplet, and is therefore analogous to superallowed Fermi (SF) transitions in nuclear beta decay. Due to its simplicity and accuracy, the theory of Fermi beta decays is one of the most precise components of the Standard Model (SM) of electroweak interactions.

The CVC hypothesis [1, 2] and quark-lepton universality relate the rate of pure vector beta decay (both pion and nuclear) to that of muon decay via the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix element V_{ud} [3, 4]. Including loop corrections, δ, the rate of pion beta decay is given by [5, 6]:

$$\Gamma_{\pi\beta} = \frac{G_\mu^2 |V_{ud}|^2}{30\pi^3} \left(1 - \frac{\Delta}{2M_+}\right)^3 \Delta^5 f(\epsilon, \Delta)(1 + \delta), \quad (1)$$

where G_μ is the Fermi weak coupling constant, $\Delta = M_+ - M_0$, $\epsilon = (m_e/\Delta)^2$, M_+, M_0, and m_e are the masses of the π^+, π^0, and the electron, respectively, while f, the Fermi function, is given by

$$f(\epsilon, \Delta) = \sqrt{1 - \epsilon}\left[1 - \frac{9}{2}\epsilon - 4\epsilon^2 \right. \quad (2)$$
$$\left. + \frac{15}{2}\epsilon^2 \ln\left(\frac{1 + \sqrt{1 - \epsilon}}{\sqrt{\epsilon}}\right) - \frac{3}{7}\frac{\Delta^2}{(M_+ + M_0)^2}\right].$$

The main experimental source of uncertainty in $\Gamma_{\pi\beta}$ amounts to just 0.05%; it comes from the measurement of Δ [7]. The combined radiative and short-range physics corrections amount to $\delta \simeq 0.033$ and are exceptionally well controlled, yielding an overall theoretical uncertainty of $\Gamma_{\pi\beta}$ of $\lesssim 0.1\%$ [6, 8, 9, 10]. Hence, pion beta decay presents an excellent means for a precise experimental determination of the CKM matrix element V_{ud}, hindered only by the low branching ratio of the decay.

The CKM quark mixing matrix has a special significance in modern physics as a cornerstone of a unified description of the weak interactions of mesons, baryons and nuclei. In a universe with three quark generations the 3×3 CKM matrix must be unitary, barring certain classes of hitherto undiscovered processes not contained in the Standard Model. Thus, an accurate experimental evaluation of the CKM matrix unitarity provides an independent check of possible deviations from the SM. As the best studied element of the CKM matrix, V_{ud} plays an important role in all tests of its unitarity. However, evaluations of V_{ud} from neutron decay have, for the most part, not been consistent with results from nuclear SF decays [11]. Clearly, a precise evaluation of V_{ud} from pion beta decay, the theoretically cleanest choice, is of interest.

The most precise measurement of the pion beta decay rate on record was made by McFarlane et al., at LAMPF by detecting in-flight π^+ decays in the LAMPF π^0 spectrometer [12]. This work reported $\Gamma_{\pi\beta} = 0.394 \pm 0.015\,\text{s}^{-1}$, which is an order of magnitude less precise than the theoretical description of the same process. Hence, we initiated the PIBETA experiment, a program of precise measurements of the rare pion and muon decays at rest, chief among them being the pion beta decay, at the Paul Scherrer Institute (PSI), Switzerland [13].

In this Letter we present an analysis of the $\pi^+ \rightarrow \pi^0 e^+ \nu$ decay events recorded with the PIBETA appara-
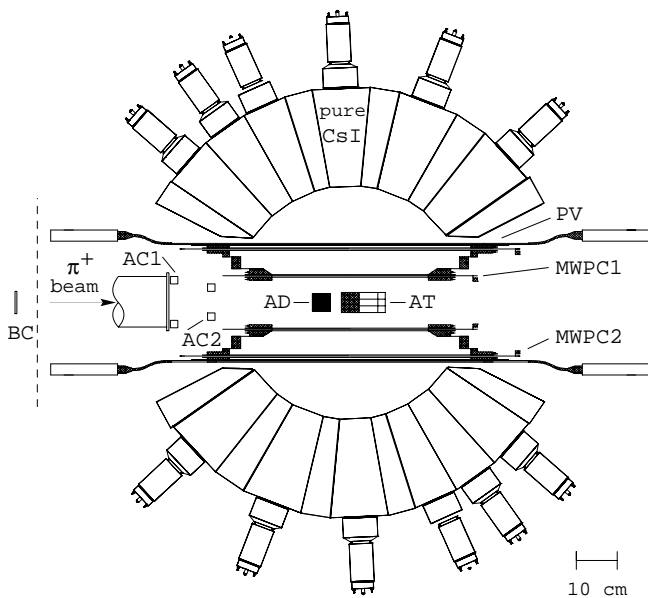
FIG. 1: A schematic cross section of the PIBETA detector system. Symbols denote: BC–thin upstream beam counter, AC1,2–active beam collimators, AD–active degrader, AT–active target, MWPC1,2–thin cylindrical wire chambers, PV–thin 20-segment plastic scintillator barrel. BC, AC1, AC2, AD and AT detectors are also made of plastic scintillator.

minimum-bias sets, we extract the π_β and π_{e2} event sets, the latter for branching ratio normalization. In a stopped pion experiment these two channels have nearly the same detector acceptance, and have much of the systematics in common.

A full complement of twelve fast analog triggers comprising all relevant logic combinations of one- or two-arm, low- or high calorimeter threshold (labeled HT and LT, respectively), prompt and delayed (with respect to π^+ stop time), as well as a random and a three-arm trigger, were implemented in order to obtain maximally comprehensive and unbiased data samples.

Signal definition and accurate counting of the π_{e2} events for normalization present a major challenge in this work. As in all previous studies, our π_{e2} data include undiscriminated soft-photon $\pi_{e2\gamma}$ events. Due to positron energy straggling in the target, accidental coincidences of multiple muon decay events, and the calorimeter energy resolution function, the π_{e2} events are superimposed on a non-negligible muon decay background. This background was removed by fitting the measured e^+ timing spectra with the functions for pion decay (signal), muon decay (background), plus the associated pile-ups (see Fig. 2 top). We also extracted the absolute π_{e2} branching ratio using this method and normalizing to the number of pion stops in the target. The results were in agreement with

tus from 1999 to 2001. We tuned the πE1 beam line at PSI to deliver $\sim 10^6$ π^+/s with $p_\pi \simeq 113\,\mathrm{MeV/c}$. The pions were slowed in an active degrader detector (AD) and stopped in a segmented 9-element active target (AT), both made of plastic scintillator. The major detector systems are shown in a schematic drawing in Fig. 1. Energetic charged decay products are tracked in a pair of thin concentric multiwire proportional chambers (MWPCs) and a thin 20-segment plastic scintillator barrel veto detector (PV). Both neutral and charged particles deposit most (or all) of their energy in a spherical electromagnetic shower calorimeter consisting of 240 elements made of pure CsI. The entire detector system, its response to positrons, photons and protons, energy and time resolution, signal definitions, along with other relevant details of our experimental method, are described at length in Ref. [14].

The measurement relies on detecting the $\pi^0 \rightarrow \gamma\gamma$ decay which immediately follows a pion beta decay event. The two photons are emitted nearly back-to-back, with about 67 MeV each. Thus, the experiment is set to record all large-energy (above the $\mu \rightarrow e\nu\bar{\nu}$ endpoint) electromagnetic shower pairs occurring in opposing detector hemispheres during a $\sim 180\,\mathrm{ns}$ long "pion gate", πG (non-prompt two-arm events). The πG is timed so as to include a sample of pile-up events preceding the pion stop. In addition, we record a large prescaled sample of non-prompt single shower (one-arm) events. Using these
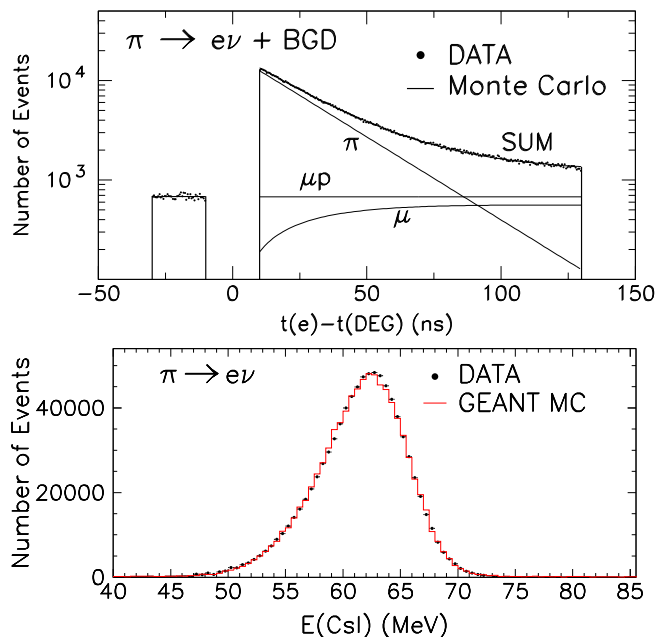

FIG. 2: Top panel: A typical histogram of time differences between the beam pion stop, $t(\mathrm{DEG})$, and 1-arm HT event time, $t(e)$, (dots), compared with a sum of the Monte Carlo-simulated responses for π_{e2} decay (π), muon decay (μ), and muon pile-up events (μp). The π_{e2} pile-up background, being much lower, is off scale in the plot. Prompt events are suppressed. Bottom panel: CsI calorimeter energy spectrum for the π_{e2} decay events, after background subtraction.
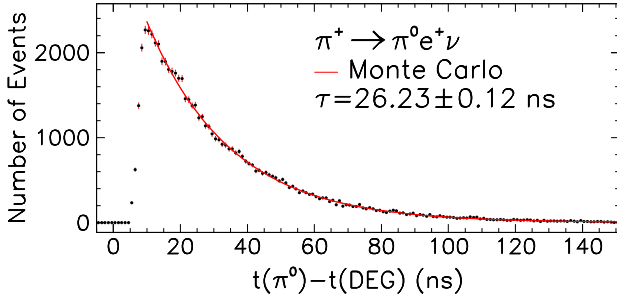
FIG. 3: Histogram of time differences between the beam pion stop and the π_β decay events (dots); curve: pion lifetime.

the recommended Particle Data Group (PDG) value [11] at a sub-percent level, with the uncertainty dominated by the systematics of the stopped pion counting. The latter is absent in our determination of $R_{\pi\beta}$. The π_{e2} energy spectrum after background subtraction is given in Fig. 2 bottom. The statistical uncertainty of the extracted number of π_{e2} events, $N_{\pi e2}$, is negligible.

The π_β signal definition was more straightforward, as seen in Figs. 3 and 4, which show the pion decay time spectrum and γ-γ relative timing histogram, respectively, for π_β events, both free of backgrounds. Finally, the histogram of recorded γ-γ opening angles for pion beta events, shown in Fig. 5, provides a sensitive test of the systematics related to the geometry of the beam pion stopping distribution, an important contributor to the acceptance uncertainty.

The π_β branching ratio $R_{\pi\beta}$ was evaluated from

$$ R_{\pi\beta} = \frac{N_{\pi\beta}}{N_{\pi^+} \, f_{\pi G} \, A_{\pi\beta}^{\mathrm{HT}} \, \tau_l \, f_{\mathrm{CPP}} \, f_{\mathrm{D}} \, f_{\mathrm{ph}}} \,, \qquad (3) $$

where $N_{\pi\beta}$ is the number of detected π_β events, N_{π^+} is the number of the decaying π^+'s, $f_{\pi G}$ is the delayed pion gate fraction, $A_{\pi\beta}^{\mathrm{HT}}$ is the HT detector acceptance evaluated by GEANT simulation, τ_l is the detector live time, f_{CPP} is the correction due to the charged particle (CP) veto system pile-up, f_{D} is the π^0 Dalitz decay correction, and f_{ph} is the photonuclear absorption correction.
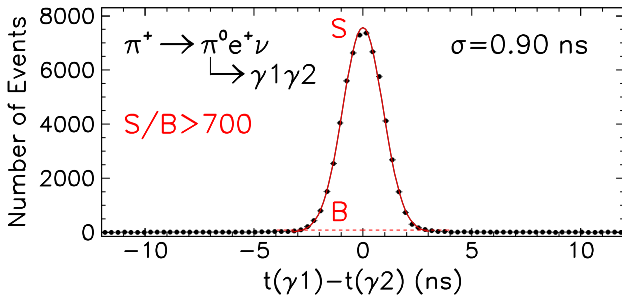

FIG. 4: Histogram of γ-γ time differences for π_β decay events (dots); curve: fit with a Gaussian function plus a constant.
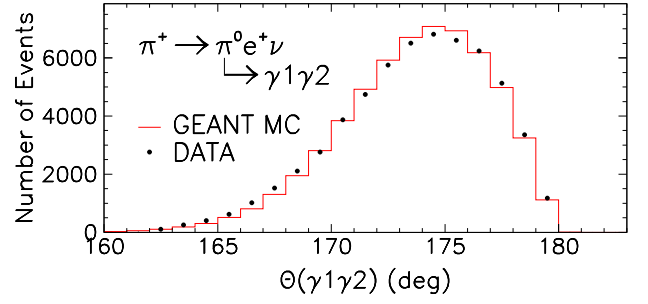
FIG. 5: Histogram of the γ-γ opening angle in π_β decay.

The $\pi \to e\nu$ branching ratio $R_{\pi e2}$ is given by

$$ R_{\pi e2} = \frac{N_{\pi e2} \, p_{\pi e2}}{N_{\pi^+} \, f_{\pi G} \, A_{\pi e2}^{\mathrm{HT}} \, \tau_l \, \epsilon_{\mathrm{PV}} \, \epsilon_{\mathrm{C1}} \, \epsilon_{\mathrm{C2}}} \,, \qquad (4) $$

where $p_{\pi e2}$ is the prescaling factor applied to π_{e2} triggers, $A_{\pi e2}^{\mathrm{HT}}$ is the high-threshold detector acceptance for $\pi \to e\nu$ decay events, including radiative corrections, while ϵ_{PV}, ϵ_{C1}, and ϵ_{C2} denote the plastic veto and wire chamber efficiencies, respectively. Clearly, taking the ratio $R_{\pi\beta}/R_{\pi e\nu}$ leads to cancellations of many common factors, apart from small corrections taking into account slight differences in thresholds, trigger timing (two-arm vs. one-arm), weighting of the efficiencies, and similar effects. Most importantly, N_{π^+}, the number of stopped pions drops out. The main sources of uncertainty are listed with their values in Table I.

As the external systematic uncertainties are self-explanatory, we turn to the internal ones. The systematic uncertainty in $N_{\pi e2}$ comes from the muon-decay background subtraction discussed above, and reflects the propagated error limits of the method. The precision of $A_{\pi\beta}^{\mathrm{HT}}/A_{\pi e2}^{\mathrm{HT}}$ is dominated by the uncertainty of the x-y-z distribution of pion stops in the target. The latter was determined with better than $50\,\mu$m accuracy by tomo-

TABLE I: Summary of the main sources of uncertainty in the extraction of $R_{\pi\beta}$, given in % (see text for discussion).

Type	Quantity	Value	Unc. (%)
external:	$\Gamma(\pi_{e2})$	1.230×10^{-4}	0.33
	$\Gamma(\pi^0 \to \gamma\gamma)$	0.9880	0.03
	π^+ lifetime	26.033 s	0.02
combined external uncertainties:			0.33
internal:	$N_{\pi e2}$ systematic	6.779×10^8	0.19
	$A_{\pi\beta}^{\mathrm{HT}}/A_{\pi e2}^{\mathrm{HT}}$	0.9432	0.12
	$r_{\pi G} = f_{\pi G}^{\pi\beta}/f_{\pi G}^{\pi e2}$	1.130	0.26
	π_β accid. bgd.	0.00	< 0.1
	f_{CPP} correction	0.9951	0.10
	f_{ph} correction	0.9980	0.10
combined internal uncertainties:			0.38
statistical:	$N_{\pi\beta}$	64,047	0.395

graphic back-tracing of π_{e2} and muon decay positrons into the target [15]. Corrections due to the undetected low portions of the e and γ energy spectra in the calorimeter (the energy "tail") contribute weakly to the acceptance uncertainty due to strong correlations between the energy responses to the two decay channels. This experiment has a unique advantage over its predecessors: it measures branching ratios as well as differential angular and energy distributions of decay products for all rare pion and muon decays simultaneously. This provides multiple redundant consistency checks of the evaluated and simulated acceptances (cf., e.g., Ref. [16]). In the present analysis the largest internal contribution to the systematic uncertainty comes from the ratio of gate fractions, $r_{\pi G}$, due to our decision to include even the earliest π_β decay events, thus maximizing the useful event statistics. An alternative method involves a sharp cut at, say, 10 ns after the pion stop time, which reduces the systematic error, along with the π_β event statistics, yielding, however, a consistent $R_{\pi\beta}$. The inherent resolution in the zero time point is excellent—it relies on the prompt $A(\pi^+, \pi^0)B$ signal and the accelerator rf pulse, providing timing calibration at the level of ~ 20 ps or better, and room for further improvement of the $r_{\pi G}$ precision. The pile-up correction $f_{\rm CPP}$ was evaluated using a random trigger, and confirmed by simulations. We modified our GEANT3 code to calculate the photonuclear correction $f_{\rm ph}$, and conservatively assigned it a 50 % uncertainty (details are given in Ref. [17]). Efficiencies $\epsilon_{\rm PV}$, $\epsilon_{\rm C1}$, and $\epsilon_{\rm C2}$, not listed in Table I, were measured with an accuracy of 0.01 % [15].

Using the above method and the PDG 2004 recommended value of $R_{\pi e2}^{\rm exp} = 1.230(4) \times 10^{-4}$ [11], we extract the pion beta decay branching ratio, our main result:

$$R_{\pi\beta}^{\rm exp} = [1.036 \pm 0.004\,({\rm stat}) \pm 0.005\,({\rm syst})] \times 10^{-8}, \quad (5)$$

or, in terms of the decay rate,

$$\Gamma_{\pi\beta}^{\rm exp} = [0.3980 \pm 0.0015({\rm stat}) \pm 0.0019({\rm syst})]\,{\rm s}^{-1}, \quad (6)$$

which represents a six-fold improvement in accuracy over the previous measurement [12]. Alternatively, the normalization can be tied to the theoretical value $R_{\pi e2}^{\rm the} = (1.2352 \pm 0.0005) \times 10^{-4}$ [18] which would increase the extracted $R_{\pi\beta}^{\rm exp}$ by 0.42 % to 1.040×10^{-8}. In a direct evaluation of the pion beta decay branching ratio using Eq (3), i.e., normalizing to the number of beam pion stops, we obtain $R_{\pi\beta} \cdot 10^8 = 1.042 \pm 0.004({\rm stat}) \pm 0.010({\rm syst})$, consistent with our main result given in Eq. (5).

Whether scaled to the experimental or theoretical $R_{\pi e2}$, our result for $R_{\pi\beta}^{\rm exp}$ is in excellent agreement with predictions of the SM and CVC given the PDG recommended value range for V_{ud} [11]:

$$R_{\pi\beta}^{\rm SM} = (1.038 - 1.041) \times 10^{-8} \quad (90\%\ {\rm C.L.}), \quad (7)$$

and represents the most accurate test of CVC and Cabibbo universality in a meson to date. Our result confirms the validity of the radiative corrections for the process at the level of $4\sigma_{\rm exp}$, since, excluding loop corrections, the SM would predict $R_{\pi\beta}^{\rm no\ rad.\ corr.} = (1.005 - 1.007) \times 10^{-8}$ at 90 % C.L.

Using our result, Eq. (5), we can calculate a new value of V_{ud} from pion beta decay, $V_{ud}^{\rm (PIBETA)} = 0.9728(30)$, which is in excellent agreement with the PDG 2004 average, $V_{ud}^{\rm (PDG'04)} = 0.9738(5)$. We will continue to improve the overall accuracy of the π_β decay branching ratio to ~ 0.5 % by further refining the experiment simulation and analysis, and by adding new data.

* Corresponding author:pocanic@virginia.edu
† Corresponding author:frlez@virginia.edu
‡ Presently at: Philips Semiconductors AG, CH-8045 Zürich, Switzerland
[1] S. S. Gershtein and I. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **29**, 698 (1955), [Soviet Phys.-JETP **2**, 576 (1956)].
[2] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[3] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[4] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[5] G. Källèn, *Elementary Particle Physics* (Addison-Wesley, Reading, Mass., 1964).
[6] A. Sirlin, Rev. Mod. Phys. **50**, 573 (1978), [erratum: *ibid.*, **50**, 905 (1978)].
[7] J. F. Crawford et al., Phys. Rev. D **43**, 46 (1991).
[8] A. Sirlin, Nucl. Phys. **B196**, 83 (1982).
[9] W. Jaus, Phys. Rev. D **63**, 053009 (2001).
[10] V. Cirigliano, M. Knecht, H. Neufeld, and H. Pichl, Eur. Phys. J. C **27**, 255 (2003).
[11] S. Eidelman et al. (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[12] W. K. McFarlane et al., Phys. Rev. D **32**, 547 (1985).
[13] D. Počanić et al., *PSI R-89.01 Experiment Proposal*, Paul Scherrer Institute, Villigen (1992).
[14] E. Frlež et al., Nucl. Inst. & Meth. in Phys. Res. A **526**, 300 (2004).
[15] W. Li, Ph.D. thesis, University of Virginia (2004).
[16] E. Frlež et al. (2003), hep-ex/0312025.
[17] $\pi\beta$ home page, http://pibeta.phys.virginia.edu.
[18] W. J. Marciano, Phys. Rev. Lett. **71**, 3629 (1993).